03 Modern Physics

Modern Physics-1 [Dual Nature of Radiation and Matter]

Dual Nature of Light

Experimental phenomena of light reflection, refraction, interference, diffraction are explained only on the basis of wave theory of light. These phenomena verify the wave nature of light.

Experimental phenomena of light photoelectric effect and Compton effect, pair production and pair annihilation can be explained only on the basis of the particle nature of light. These phenomena verify the particle nature of light. it is inferred that light does not have any definite nature, rather its nature depends on its experimental phenomenon. This is known as the dual nature of light. The wave nature and particle nature both cannot be possible simultaneously.

Electron Emission

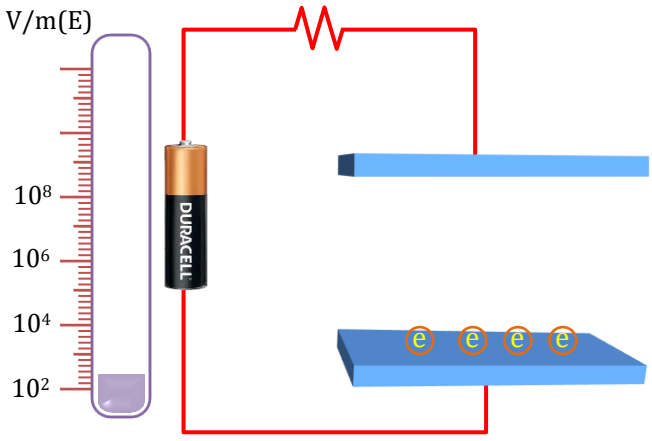
- Emission of electron from a metal surface, when energy is incident upon the surface.
- Minimum energy required to bring out the electron from metal surface is called work function (ϕ_0).
- It depends on nature of material and nature of surface.

Types of Electron Emission

1. Thermionic emission 2. Field emission 3. Photoelectric emission

1. Thermionic emission : The phenomenon of emission of electrons from the metal surface when it is heated is called thermionic emission.

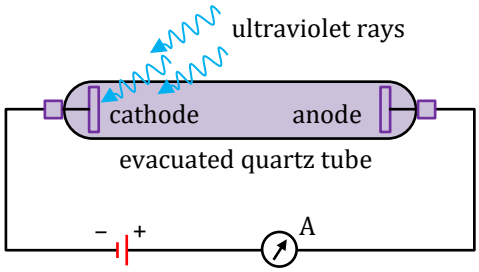
2. Field emission : In this process strong electric field ($\approx 10^8$ V/m) is applied to a metal, and the electrons are emitted from the surface.



3. Photoelectric emission: When light of suitable frequency illuminates a metal surface, electrons are emitted from the metal surface. This phenomenon is called photo electric emission.

Different Experiments

Hertz Experiment: Hertz observed that when ultraviolet rays are incident on a negative plate of electric discharge tube then conduction takes place easily in the tube.



Hallwach Experiment

Hallwach observed that if negatively charged Zn plate of electroscope is illuminated by ultra violet light, its negative charge decreases and becomes neutral and after some time, it gains positive charge. It indicates that under the action of ultra violet light, some negative charged particles are emitted from the metal.

Lenard Experiment

He told that when ultraviolet rays are incident on cathode, electrons are ejected, these electrons are attracted by anode and due to complete path of photo electrons, photo current flows. When ultra violet rays are incident on anode, electrons are ejected but current does not flow.

Work Function (ϕ)

It is the minimum energy required by an electron to escape from metal surface.

Work function depends on the nature of metal and its surface.

Minimum : Cs = 2.14 eV ; Maximum : Pt = 5.65 eV $\quad (1eV = 1.6 \times 10^{-19}\ J)$

Einstein's Quantum theory of Light

- Light behaves as Quanta, when it interacts with the matter.
  These energy quanta are called photons, having definite Energy and Momentum.
- Photons always travel with speed of light in vacuum. $(c = 3 \times 10^8\ m/s)$
- Photons are electrically neutral and are not affected by electric and magnetic fields.

Energy of photon

Energy radiated from a source propagates (microscopically) in the form of small packets and these are known as photons. according to Planck the energy of a photon is directly proportional to the frequency of the radiation.

$E \propto \nu \Rightarrow E = h\nu$

$E = \frac{hc}{\lambda} \quad (\because c = \nu\lambda) \quad \Rightarrow \quad E = \frac{hc}{\lambda} = \frac{12400}{\lambda}\ eV - Å \quad [\because hc = 12400\ (Å - eV)]$

Here; E = energy of photon, c = speed of light, h = Planck's constant ($h = 6.62 \times 10^{-34}$ J-s),
e = charge of electron, ν = frequency of photon, l = wavelength of photon

Linear momentum of photon

Linear momentum of photon $p = \frac{E}{c} = \frac{h\nu}{c} = \frac{h}{\lambda}$

Effective mass of photon

Effective mass of photon $m = \frac{E}{c^2} = \frac{hc}{c^2\lambda} = \frac{h}{c\lambda}$ i.e. effective mass $m \propto \frac{1}{\lambda}$

So, mass of violet light photon is greater than the mass of red light photon. ($\because \lambda_R > \lambda_V$)

Rest mass of a photon is always zero.

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Intensity of light

$I = \frac{E}{At} = \frac{P}{A}$ ...(i)

SI Unit : $\frac{\text{joule}}{m^2 - s}$ or $\frac{\text{watt}}{m^2}$

Here; P = power of source, A = Area, t = time taken

E = energy incident in t time = Nhν, N = number of photon incident in t time

Intensity $I = \frac{N(h\nu)}{At} = \frac{n(h\nu)}{A}$ ...(ii) $\left[\because \quad n = \frac{N}{t} = \text{no. of photon per sec.}\right]$

from equation (i) and (ii), $\frac{P}{A} = \frac{n(h\nu)}{A}$

$n = \frac{P}{h\nu} = \frac{P\lambda}{hc} \quad \Rightarrow \quad n = (5 \times 10^{24}\ J^{-1}\ m^{-1})\ P \times \lambda$

Illustration 1:

Power of a radiator is 100 watt and its λ is 400 nm. Calculate No. of photon emitted in 10 hrs.

Solution:

$n = (5 \times 10^{24}\ J^{-1}\ m^{-1})\ P \times \lambda$

$n = 5 \times 10^{24} \times 100 \times 4 \times 10^{-7} = 2 \times 10^{20}$

$N = n \times t = 2 \times 10^{20} \times 10 \times 3600 = 7.2 \times 10^{24}$

Illustration 2:

A source S_1 is producing 10^{15} photon/s of wavelength 5000 Å. Another source S_2 is producing 1.02×10^{15} photon/s of wavelength 5100 Å. Calculate ratio of power of S_2 to power of S_1.

Solution:

$n = \frac{P\lambda}{hc}$ or $P = \frac{nhc}{\lambda}$

$\frac{P_2}{P_1} = \frac{n_2}{\lambda_2} \times \frac{\lambda_1}{n_1} = \frac{1.02 \times 10^{15} \times 5000}{5100 \times 10^{15}} = \frac{1}{1}$

Illustration 3:

In an photo-electric experiment frequency of incident light is doubled and its intensity is trippled. How much time the photon current is obtained?

Solution:

$I = nh\nu \quad ; \quad n = \frac{3}{2}$ times

$i \propto n$

So, $i = \frac{3}{2}$ times

Illustration 4:

The energy flux of sunlight reaching the Earth's surface is $1.388 \times 10^3\ Wm^{-2}$. How many photons (nearly) per square meter are incident on the earth per second? Assume that the photons in the sunlight have an average wavelength of 1550 nm.

Solution:

$n = \frac{IA}{E} = \frac{IA\lambda}{hc}$

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Illustration 5:

The power of a bulb is 60 milliwatt and the wavelength of light is 6000 Å. Calculate the number of photons/second emitted by the bulb?

Solution:

Energy released per second

$$nh\nu = \text{power} \Rightarrow \frac{nhc}{\lambda} = P \Rightarrow n = \frac{P\lambda}{hc} = \frac{60\times10^{-3}\times6000\times10^{-10}}{6.62\times10^{-34}\times3\times10^{8}} \text{ photon/sec} = 1.8 \times 10^{17} \text{ photon/sec}$$

Illustration 6:

The energy flux of sunlight reaching on the Earth's surface is 1.388×10^3 W/m^2. How many photons (nearly) per square metre are incident on the Earth per second? Assume that the photons in the sunlight have an average length of 550 nm.

Solution:

$$\text{Intensity, } I = \frac{\text{Power emitted}}{\text{Area}} = \frac{P}{A} = 1.388 \times 10^3 \text{ W/m}^2$$

Also, Energy of a photon, $E = \frac{hc}{\lambda} = \frac{6.63\times10^{-34}\times3\times10^{8}}{550\times10^{-9}} = 3.616 \times 10^{-19}$ J

Let n be the total number of photon/area then, $n = \frac{P/A}{E} = \frac{I}{E} = \frac{1.388\times10^{3}}{3.616\times10^{-19}} = 3.83 \times 10^{21}$ photon/m^{2}s

Radiation Force and Pressure

(i) When radiations are falling normally on a perfectly reflecting surface -

Let 'N' photons are falling in time t,

momentum before striking the surface $(p_1) = \frac{Nh}{\lambda}$

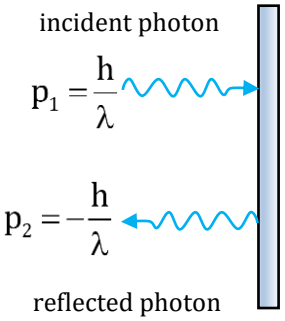
momentum after striking the surface $(p_2) = -\frac{Nh}{\lambda}$

change in momentum of photons = $p_2 - p_1 = \frac{-2Nh}{\lambda}$

momentum transferred to the surface = $\Delta p = \frac{2Nh}{\lambda}$

radiation force on the surface $F = \frac{\Delta p}{\Delta t} = \frac{2Nh}{t\lambda} = n\left[\frac{2h}{\lambda}\right]$ but $n = \frac{P\lambda}{hc}$

$\therefore \quad F = \frac{2h}{\lambda}\times\frac{P\lambda}{hc} = \frac{2P}{c}$ and Radiation pressure = $\frac{F}{A} = \frac{2P}{cA} = \frac{2I}{c} \quad \left[\because I = \frac{P}{A}\right]$



(ii) When radiations are falling normally on a perfectly absorbing surface –

Radiation force $F=\frac{p_1-p_2}{t}=\frac{\frac{Nh}{\lambda}-0}{t}=\frac{Nh}{t\lambda}\Rightarrow F=n\frac{h}{\lambda}$ $\left[\because n=\frac{P\lambda}{hc}\right]$

$F=\frac{P}{c}$ and Pressure = $\frac{F}{A}=\frac{P}{Ac}=\frac{I}{c}$

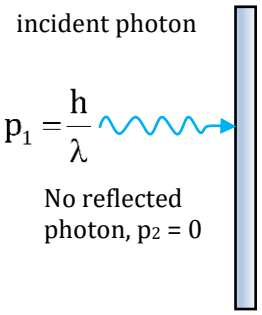


Illustration 7:

A beam of electromagnetic wave of power 10 watt is incident normally on a surface, which absorb 40% power and remaining power is reflected. Calculate force exerted on a surface.

Solution:

$F=\frac{P_i+P_r}{c}=\frac{10+6}{3\times10^8}=\frac{16}{3\times10^8}=5.33\times10^{-8}N$

Illustration 8:

A 100 W sodium lamp radiates energy uniformly in all directions. The lamp is located at the centre of a large sphere that absorbs all the sodium light, which is incident on it. The wavelength of the sodium light is 589 nm. (a) What is the energy per photon associated with the sodium light?

(b) At what rate are the photons delivered to the sphere? [NCERT]

Solution:

(a) $E=\frac{hc}{\lambda}$

(b) $n=\frac{P\lambda}{hc}=\frac{P}{E}$

Illustration 9:

How many photons of wavelength λ = 6600 nm must strike a totally reflecting screen per second at normal incidence so as to exert a force of 1N ?

Solution:

Momentum of the incident photon, $p=\frac{h}{\lambda}$, Momentum after reflection = $-\frac{h}{\lambda}$

Change in momentum = $\Delta p=\frac{2h}{\lambda}$

If n is the number of photons falling per second on the screen, then force

$F=\frac{\Delta p}{\Delta t}=\frac{2nh}{\lambda}\Rightarrow n=\frac{F\lambda}{2h}=\frac{1\times6600\times10^{-9}}{2\times6.6\times10^{-34}}=5\times10^{27}$ photons s^{-1}

BEGINNER'S BOX-1

1. Who discovered photo electric effect ?
(1) Hertz (2) Lenard (3) Hallwach (4) Einstein

2. The energy of a photon is equal to 3 kilo eV. Calculate its linear momentum ?

3. A parallel beam of monochromatic light of wavelength 500 nm is incident normally on a perfectly absorbing surface. The power through any cross-section of the beam is 10 W. Find
(a) Number of photon absorbed by the surface per second.
(b) The force exerted by light beam on the surface.

4. Which colour of photon has greater energy either red or violet ?

5. A TV station is operated at 100 MW with a signal frequency of 10 MHz. Calculate the number of photons radiated per second by its antenna.

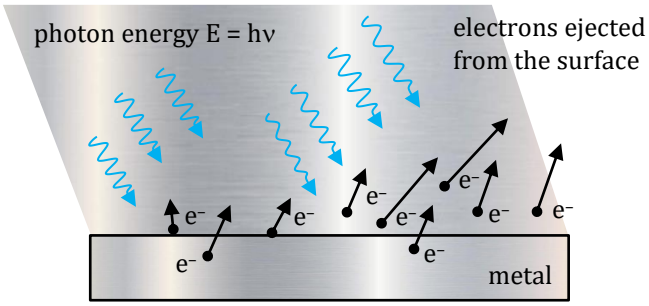
6. Calculate number of photons passing through a ring of unit area in unit time if light of intensity 100 Wm^{-2} and of wavelength 400 nm is falling normally on the ring.

7. A special kind of light bulb emits monochromatic light of wavelength 700 nm. Electrical energy supply to it at the rate of 60 W and the bulb is 50% efficient at converting that energy to light energy. How many photons are emitted by the bulb during its life time of 1 day.

Photoelectric Effect

It is a phenomenon of ejecting electrons by falling light of suitable frequency or suitable wavelength on a metal. Ejected electron are called photoelectrons and current flowing due to the photoelectrons is called photoelectric current. This effect was discovered by Hertz.

Laws of photo electric effect were given by Lenard. It was explained by Einstein using Quantum theory of light.



The work function (ϕ) represents the energy needed to remove the least tightly bound electrons from the surface. It depends only on nature of the metal surface and independent of any other factors.

➢ Condition for PEE ; Energy of photon $\geq \phi$

Experimental Setup of Photoelectric Effect by Lenard

When light of frequency ν and intensity I falls on the cathode, electrons are emitted from it. The electrons are collected by the anode and a current flows in the circuit. This current is called photoelectric current. This experiment is used to study the variation of photoelectric current with different factors like intensity, frequency and the potential difference between the anode and cathode.

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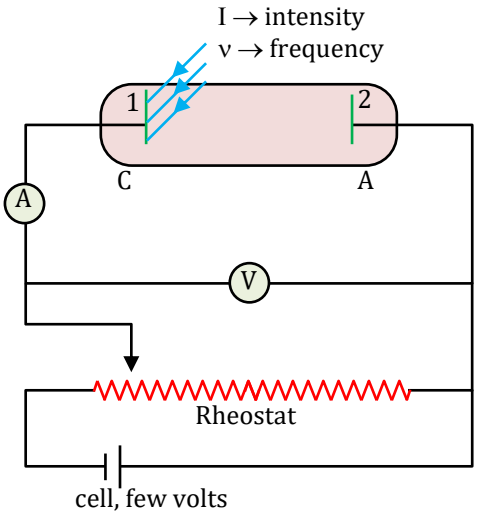


Photoelectron: The electron emitted in photoelectric effect is called photoelectron.

Photoelectric current: If current passes through the circuit in photoelectric effect then the current is called photoelectric current.

Work function: The minimum energy required to make an electron free from the metal is called work function. It is constant for a metal and denoted by ϕ or W. It is the minimum for Cesium. It is relatively less for alkali metals.

Saturation current: When all the photo electrons emitted by cathode reach the anode then current flowing in the circuit at that instant is known as saturated current, this is the maximum value of photoelectric current.

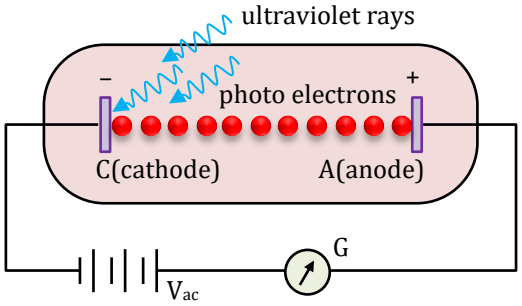
Stopping potential: Minimum magnitude of negative potential of anode with respect to cathode for which current is zero is called stopping potential. This is also known as cutoff voltage. This voltage is independent of intensity.

Retarding potential: Negative potential of anode with respect to cathode which is less than stopping potential is called retarding potential.

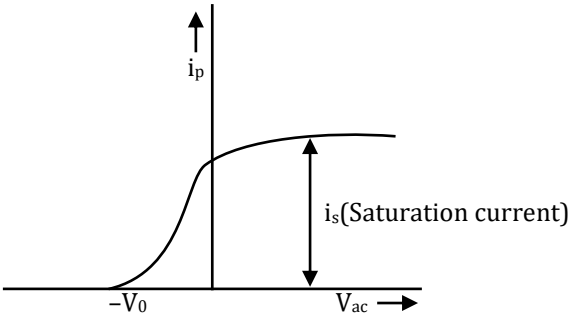
Observations of Lenard's Experiment of Photoelectric Effect

The effect of potential difference between A and C: Light of fixed frequency (ν) and intensity (I) is incident on photo emissive plate C, keeping A positive w.r.t. C. If positive potential of A gradually increased, it is found that first photo current increases then becomes maximum called saturation current.

Now we apply negative potential to plate A w.r.t. C and on increasing it gradually, current decreases and becomes zero at acertain negative potential.



Magnitude of the minimum negative potential at which i_p becomes zero, called stopping potential (V_0).



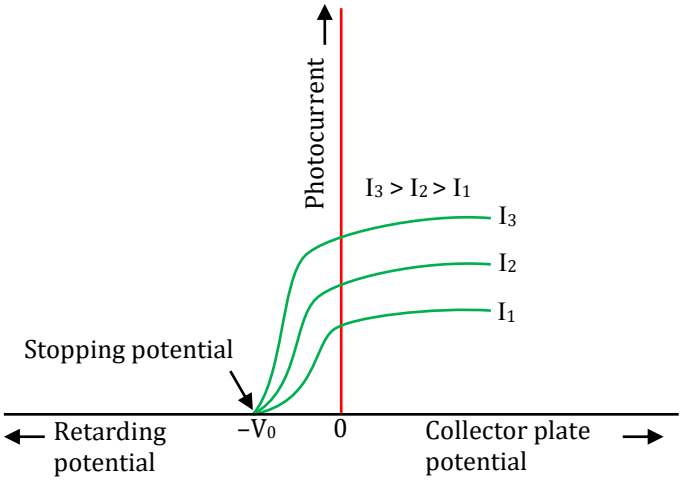
At stopping potential most energetic electrons are stopped.

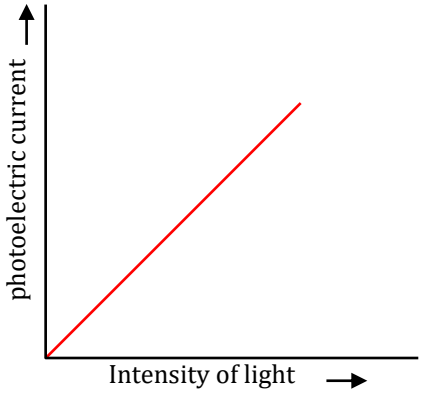
It means stopping potential measures maximum kinetic energy of photo electrons. i.e. $K_{max.} = eV_0$

Stopping potential does not depend on the distance between emitter and collector.

Effect of intensity of light

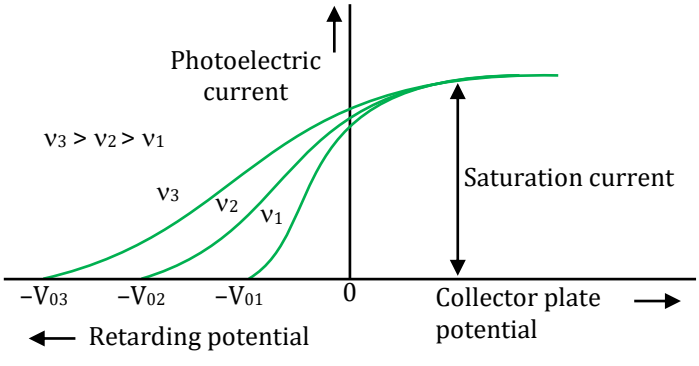
When intensity of given source able to produce PEE, is increased, i_s increases but V_0 remains unchanged.

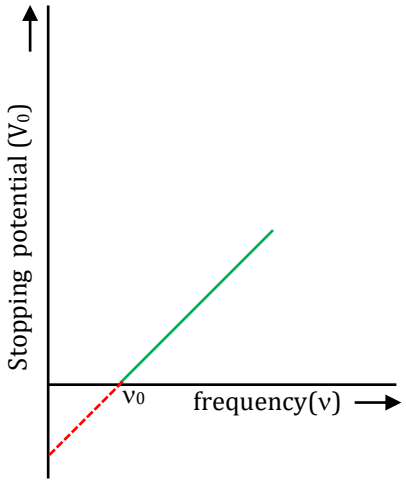




Effect of frequency

When frequency of radiations is increased, V_0 increases but i_s is almost unchanged. It is also observed that below a certain frequency photoelectrons don't come out. The minimum frequency which can eject the photo electrons is called cut off or threshold frequency. Similarly corresponding maximum wavelength is called threshold wavelength.





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Time lag

There is no time lag between the incidence of radiations and emission of electrons. The photoelectric emission is an instantaneous process without any apparent time lag ($\sim 10^{-9}$ s or less), even when the incident radiation is made exceedingly dim.

Einstein's Photoelectric Equation

Radiations absorbed by the surface are in the form of quanta (photon). Energy of each photon depends on frequency. One photon can interact with one electron at a time. In the interaction between photon and electron incident photon transfers its whole energy to the electron.

If energy is sufficient then electron comes out without any time delay. It means phot electric effect is an instantaneous process.

If intensity of the given source is increased then number of photon increases. So that, more number of electrons are emitted and greater saturation current is obtained. It means saturation current depends upon intensity of the given source, $i_s \propto I$

At a time, only one photon can interact with one electron.

Energy of photon used by the electron is,

$h\nu$ = Kinetic energy of electron + Energy required to make electron free from the metal surface (ϕ_0) + Energy lost in collision before emission (Q)

If Q = 0, means there is no heat loss, then kinetic energy of electron is maximum.

Now, $h\nu = (K.E._{max}) + \phi_0$ It is known as Einstein's equation of P.E.E.

$\mathbf{(K.E._{max}) = h\nu - \phi_0}$ or $eV_0 = h\nu - \phi_0$ or $eV_0 = h\nu - h\nu_0$

Here, ν_0 is threshold frequency for that, $V_0 = 0$.

It means maximum K.E. and stopping potential (V_0) depends on frequency. It is independent of intensity of the given source.

Kinetic energy cannot be negative so that, $h\nu > \phi_0$

$h\nu > h\nu_0$ $\left[\text{Here} \quad \phi_0 = h\nu_0 = \frac{hc}{\lambda_0},\ \phi_0 = \frac{12400}{\lambda_0} eV - \mathring{A}\right]$

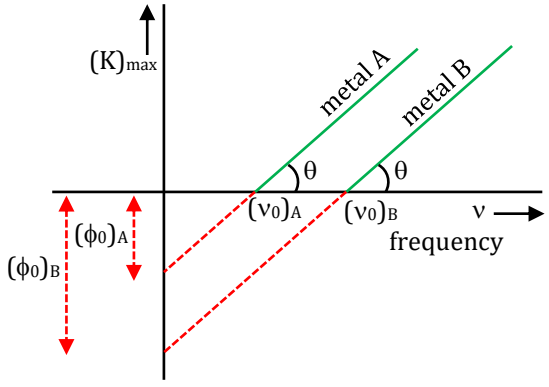
$\nu > \nu_0$ It means if frequency is less than 'ν_0', electron does not come out.

Graph between $(K.E.)_{max}$. and frequency;

$(K)_{max.} = h\nu - \phi_0$ [$\because$ Y = mx – c]

slope = m = tanθ = h (same for all metals)

$(\phi_0)_B > (\phi_0)_A$



Graph between stopping potential (V_0) and frequency (ν)

$\because \quad eV_0 = h\nu - \phi_0$

$$V_0 = \left[\frac{h}{e}\right]\nu - \left[\frac{\phi_0}{e}\right]$$

slope = m = $\tan\theta = \frac{h}{e}$ (same for all metals)

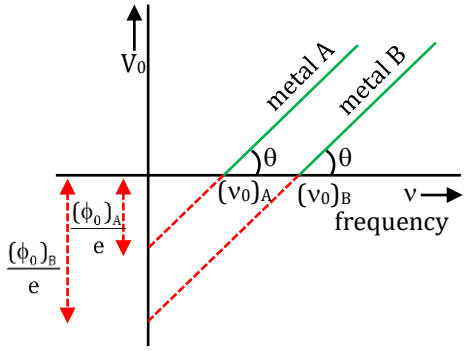


Illustration 10:

The threshold wavelength of a metal is 400 nm. Photo electrons have kinetic energy maximum 1.5 eV. Find the wavelength of incident photon.

Solution:

$\lambda_0 = 400 \text{ nm} = 4000 \text{ Å}$ $\qquad$ $K.E._{max.} = \frac{12400\,eVÅ}{\lambda} - \frac{12400\,eVÅ}{\lambda_0}$

$1.5 \text{ eV} = \frac{12400\,eVÅ}{\lambda} - \frac{12400\,eV}{4000} \Rightarrow 1.5 \text{ eV} = \frac{12400\,eVÅ}{\lambda} - 3.1 \text{ eV}$

$(1.5 + 3.1) \text{ eV} = \frac{12400\,eVÅ}{\lambda} \Rightarrow \lambda = \frac{12400Å}{4.6} \Rightarrow \lambda = 2696 \text{ Å}$

Illustration 11:

The work function of a metal is 2.3 eV and the wavelength of incident photon is 4.8×10^{-7}m. Find maximum kinetic energy of photo electrons.

Solution:

$\phi = 2.3 \text{ eV}$ and $\lambda = 4.8 \times 10^{-7} \text{ m} = 4800 \text{ Å}$

$K.E._{max.} = \frac{hc}{\lambda} - \phi = \frac{(6.62\times10^{-34}\text{Js})(3\times10^{8}\text{ms}^{-1})}{\lambda} - \phi = \frac{12400\,eVÅ}{\lambda} - \phi$

$K.E._{max.} = \left[\frac{12400}{4800} - 2.3\right] eV = 0.28 \text{ eV}$

Illustration 12:

Light quanta with an energy 4.9 eV eject photoelectrons from metal with work function 4.5 eV. Find the maximum impulse transmitted to the surface of the metal when each electron flies out.

Solution:

According to Einstein's photoelectric equation

$K_{max} = \frac{1}{2} mv^2_{max} = h\nu - \phi_0 = 4.9 - 4.5 = 0.4 \text{ eV}$

Change of momentum ($p_2 - p_1$) = impulse $\therefore$ impulse = $mv - \left(-\frac{E}{c}\right) = mv + E/c$

Maximum impulse = $\sqrt{2mK_{max}} + \frac{E}{c}$

$= \sqrt{2\times0.4\times1.6\times10^{-19}\times9.1\times10^{-31}} + \frac{4.9\times1.6\times10^{-19}}{3\times10^{8}} = 3.43\times10^{-25} \text{ kg m/s}$

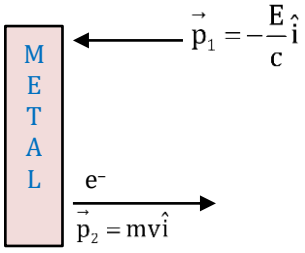


Illustration 13:

The stopping potential for the photoelectrons emitted from a metal surface of work function 1.7 eV is 10.4V. Find the wavelength of the radiation used. Also identify the energy levels in hydrogen atom which will emit this wavelength.

Solution:

Energy of radiation, $h\nu = KE_{max} + \phi_0 = eV_0 + \phi_0 = 10.4\text{ eV} + 1.7\text{ eV} = 12.1\text{ eV}$

Wavelength corresponding to this energy, $\lambda = \frac{hc}{E} = \frac{12400}{12.1\text{eV}}\text{eV–Å} = 1024\text{ Å}$

As $\Delta E = 13.6\left(\frac{1}{1^2} - \frac{1}{3^2}\right) = 12.1\text{ eV}$

So this radiation will be emitted by transition; $n = 3 \rightarrow n = 1$

Quantum Efficiency

Quantum efficiency (x) = $\frac{\text{number of electrons emitted per second}}{\text{total number of photons incident per second}} = \frac{n_e}{n_{ph}}$

If quantum efficiency is x% then $n_e = \frac{x}{100}\ n_{ph}$ [Here, $n_{ph} = (5 \times 10^{24}\ J^{-1}\ m^{-1})\ P\lambda$]

Photoelectric current

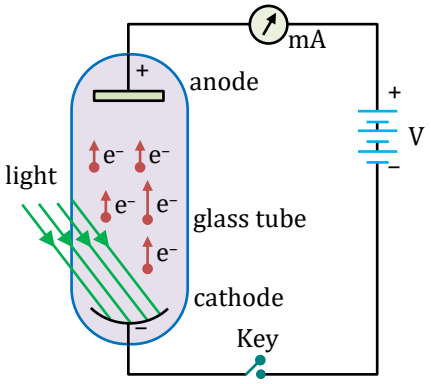
Photoelectric current, $I_e = \frac{\text{charge}}{\text{time}} = \frac{Q}{t} = n_e\ e = 1.6 \times 10^{-19}\ n_e$

Photocell

A photo cell is a practical application of the phenomenon of photo electric effect, It converts light energy into electrical energy.

Construction: A photo cell consists of an evacuated sealed glass tube containing anode and a concave cathode of suitable emitting material such as Cesium (Cs).

Working: When light of frequency greater than the threshold frequency of cathode material falls on the cathode, emitted photoelectrons are collected by the anode and an electric current starts flowing in the external circuit. The current increases with the increase in the intensity of light. The current would stop, if the light does not fall on the cathode.



Application

(i) In television camera. (ii) In automatic door (iii) Burglar's alarm

(iv) Automatic switching of street light and traffic signals.

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Failure of Wave Theory in Explanation of Photoelectric Effect

(i) According to wave theory when light incident on a surface, energy is distributed continuously over the surface. So that electron must take a time interval to accumulate sufficient energy to come out. But in experiment there is no time lag.

(ii) When intensity is increased, more energetic electrons should be emitted. So that stopping potential should be intensity dependent. But it is not observed.

(iii) According to wave theory, if intensity is sufficient then, at each frequency, electron emission is possible. It means there should not be existence of threshold frequency.

BEGINNER'S BOX-2

1. The work function of a metal is 4 eV if 5000Å wavelength of light is incident on the metal. Is there any photo electric effect ?

2. In a photo cell 4 unit photo electric current is flowing, the distance between source and cathode is 4 unit. Now distance between source and cathode becomes 1 unit. What will be photo electric current now ?

3. The wavelength of photons in two cases are 4000 Å and 3600 Å respectively what is difference in stopping potential for these two ?

4. Threshold frequency of a surface is ν_0. It is illuminated by $3\nu_0$ frequency, then maximum speed of photo electrons is V m/sec. What will be maximum speed if incident frequency is $9\nu_0$?

5. When incident wavelength is λ, stopping potential is $3V_0$. If incident wavelength is 2λ then stopping potential is V_0. Find out threshold wavelength in terms of λ.

6. A light beam of power 1.5mW and 400nm wavelength incident on a cathode. If quantum efficiency is 0.1% then, find out obtained photo current and number of photoelectron per second.

7. A metalic surface of work function $h\nu$ is illuminated by a radiation beam of frequency 5ν. Stopping potential observed is X. What will be stopping potential if the surface is illuminated by radiations of 7ν frequency?

8. The kinetic energy of the fastest moving photo electron from a metal of work function 2.8 eV is 2eV. If the frequency of light is doubled, then find the maximum kinetic energy of photo electron.

9. A monochromatic light incident on metal 'A' having threshold frequency ν_0. It emits photo electrons of maximum kinetic energy K. Now incident frequency is made three times and fall on a metal 'B' having threshold frequency $2\nu_0$. What will maximum kinetic energy of photo electrons emitted by metal 'B' ?

10. If light of wavelength 4000 Å falls on a metal which has a stopping potential 1·4 volt against photoelectric emission then what is the work function of the metal. [Take $h = 6{\cdot}6 \times 10^{-34}$ Js and $c = 3 \times 10^8$ ms^{-1}]

Matter Wave-De Broglie Hypothesis

Dual nature of light

1. Sometimes it behaves like a particle (called a photon), which explains how light travels in straight lines and interacts with matter. For example, in experiments of PEE, Compton effect, Raman effect light behaves as particle.
2. Sometimes it behaves like a wave, which explains how light show interference, diffraction and polarisation.

De-Broglie Hypothesis

Moving particles of matter should display wave like properties under suitable conditions. Because nature loves symmetry and the two basic physical entities- matter and energy, must have symmetrical character. If radiation shows dual aspects, so matter should also show dual nature (particle as well as wave).

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Introduction of matter waves

1. Waves associated with a moving particles are matter waves. (not with stationary particles).
2. Every wave has a wave function, for example EM wave has electric and magnetic field as a functions of time. Mechanical wave has medium particle's displacement as a function of time. But matter waves can not be expressed by such functions therefore it is stated that, "matter waves are neither EM waves nor mechanical waves."
3. atter waves are probabilistic waves because these are based on concept of probability and can be expressed by probability density functions.
4. Matter waves are also known as de-Broglie waves. Because their concept was given by physicist Louis Victor de Broglie.
5. Macroscopic objects in our daily life do not show wave-like properties because their wavelength are so small (beyond any measurement). But the wave character of microscopic particles (sub-atomic particles) is significant and measurable.

Formula for de-Broglie wavelength

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2mK}}$$

Where : λ = de-Broglie wavelength associated with the moving particles.

h = Planck's constant = 6.63×10^{-34} J-s = 4.132×10^{-15} eV-s

p = Linear momentum = mv

K = Kinetic energy = $\frac{1}{2}mv^2 = \frac{p^2}{2m} \Rightarrow p^2 = 2mK \Rightarrow p = \sqrt{2mK}$

De-Broglie wavelength for a charged particle

Consider a charged particle of mass 'm' and charge 'q' accelerated from rest through a potential V, then its

(a) Kinetic energy K = qV

(b) Momentum $p = \sqrt{2mK} = \sqrt{2mqV}$

(c) Velocity $v = \sqrt{\frac{2qV}{m}}$ $\left[\because K = \frac{1}{2}mv \Rightarrow v = \sqrt{\frac{2K}{m}}\right]$

(d) de-Broglie wavelength $\lambda = \frac{h}{\sqrt{2mqV}}$

Examples of different charged particles

1. Electron ($_{-1}e^0$): $m_e = 9.1 \times 10^{-31}$ kg ; $q_e = e = 1.6 \times 10^{-19}$ C

   $$\lambda_e = \frac{12.27}{\sqrt{V}}\text{Å}\sqrt{\text{volt}} = \sqrt{\frac{150}{V}}\text{Å}\sqrt{\text{volt}}$$

2. Proton ($_1p^1$): $m_p = 1.67 \times 10^{-27}$ kg ; $q_p = e = 1.6 \times 10^{-19}$ C

   $$\lambda_p = \frac{0.2863}{\sqrt{V}}\text{Å}$$

3. Deutron ($_1H^2$ or d): $m_d = m_p + m_n \approx 2m_p$; $q_d = q_p + q_n = e + 0 = e$

   $$\lambda_d = \frac{0.2021}{\sqrt{V}}\text{Å}\sqrt{\text{volt}}$$

4. Alpha particle (α or $_2He^4$) : $m_a = 2m_p + 2m_n \approx 4m_p$; $q_\alpha = 2q_p + 2q_n = 2e + 0 = 2e$

   $$\lambda_\alpha = \frac{0.1012}{\sqrt{V}}\text{Å}\sqrt{\text{volt}}$$

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Illustration 14:

With what potential an electron should be accelerated so that its de-Broglie wavelength becomes 10Å.

Solution:

$$\lambda_e = \sqrt{\frac{150}{V}}\text{Å}\sqrt{\text{volt}} = 10\text{Å} \Rightarrow V = \frac{150}{100} \text{ volt} = 1.5 \text{ volt}$$

Illustration 15:

Find the ratio of de-Broglie wavelengths for a proton and an alpha particle, if both have same speed.

Solution:

$$\lambda = \frac{h}{mv} \Rightarrow \lambda \propto \frac{1}{m}$$

$$\frac{\lambda_p}{\lambda_\alpha} = \frac{m_\alpha}{m_p} = \frac{4m_p}{m_p} = \frac{4}{1}$$

Illustration 16:

Find the ratio of de-Broglie wavelength for a proton and a deutron, if both have same kinetic energy.

Solution:

$$\lambda = \frac{h}{\sqrt{2mK}} \Rightarrow \lambda \propto \frac{1}{\sqrt{m}}$$

$$\frac{\lambda_p}{\lambda_d} = \sqrt{\frac{m_d}{m_p}} \simeq \sqrt{\frac{2m_p}{m_p}} = \frac{\sqrt{2}}{1}$$

De-Broglie Wavelength associated with Uncharged Particles

De-Broglie wavelength of neutral particles

1. Neutron ($_0n^1$):

$$\lambda_n = \frac{h}{P_n} = \frac{h}{m_n v_n} = \frac{h}{\sqrt{2m_n K}}$$ Here K must be in joule (J)

$$\lambda_n = \frac{0.2863}{\sqrt{K}}\text{Å}\sqrt{\text{eV}}$$ Here K must be in eV

Special Note: For thermal neutrons, average kinetic energy is given by $\frac{3}{2}kT$.

$$\therefore \lambda_n = \frac{h}{\sqrt{2m_n\left(\frac{3}{2}kT\right)}} = \frac{h}{\sqrt{3m_n kT}} \approx \frac{25.6}{\sqrt{T}}\text{Å}\sqrt{K}$$ (Temperature is in Kelvin)

2. Gas molecule: Total kinetic energy of any gas molecule is $\frac{f}{2}kT$ where = f = degree of freedom.

But total translatory kinetic energy of any gas molecule is $\frac{3}{2}kT$. Because de-Broglie wavelength is associated with translational motion only. Therefore, for any gas molecule

$$\lambda = \frac{h}{\sqrt{2m\left(\frac{3}{2}kT\right)}} = \frac{h}{\sqrt{3mkT}}$$ (Here T is in kelvin scale)

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Illustration 17:

Compute the typical de-Broglie wavelength of an electron in a metal at 27°C and compare it with the mean separation between two electrons in a metal, which is given to be about 2×10^{-10}m.

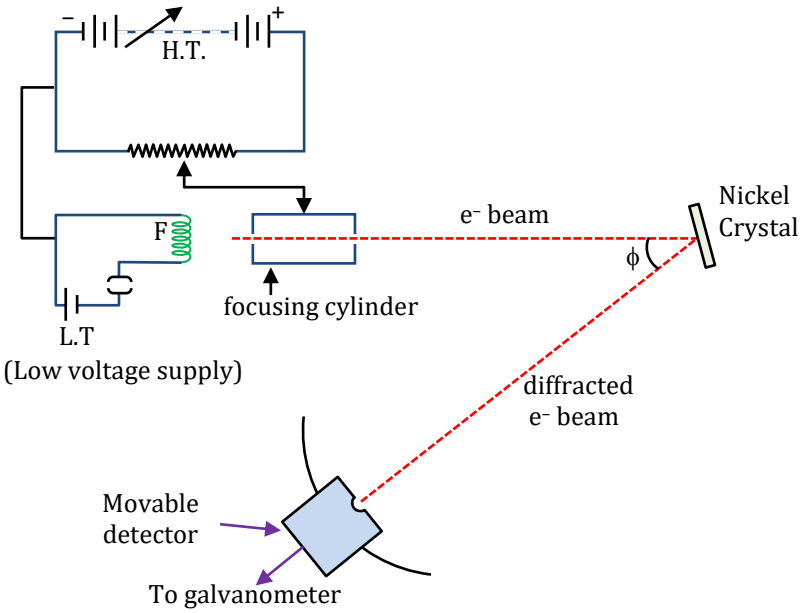
Solution:

$$\lambda = \frac{h}{\sqrt{3mkT}} = \frac{6.6\times10^{-34}}{\sqrt{3(9.1\times10^{-31})\times(1.38\times10^{-23})\times(273+27)}} = 6.2\times10^{-9}\,m$$

$$\text{As mean separation between two electrons} = \frac{r}{\lambda} = \frac{2\times10^{-10}\,m}{6.2\times10^{-9}\,m} \approx 0.03$$

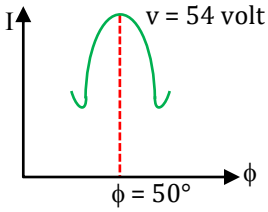
Davisson Germer Experiment

1. This experiment experimentally verifies the concept of matter waves.
2. In this experiment a sharp accelerated electron beam is diffracted by the nickel crystal.
3. Experimental set-up

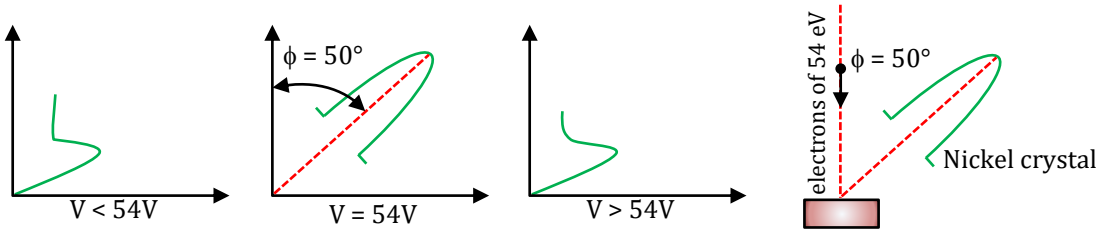


4. Basic elements used in the experimental set-up:-
   (i) **Electron gun:-** It provides a sharp beam of electrons of different K.E., which is given by K.E. = eV_{acc} Electron gun consists a tungsten filament F, coated with barium oxide. By thermionic emission, electrons are emitted by filament and accelerated to a desired velocity by applying high voltage power supply. They are made to pass through a cylinder with fine holes along its axis, producing a fine collimated beam.
   (ii) **Ni crystal:-** It is used to diffract/scatter electrons in all direction.
   (iii) **Detector:-** It is used to measure intensity of scattered electron beam in a given direction.
   The detector can be moved on a circular scale and is connected to a sensitive galvanometer whose deflection is proportional to the intensity of the electron beam entering the collector.

Conclusion and results : Curve between the intensity (I) of diffracted electrons and diffracting angle (ϕ)



Graph I versus ϕ for different accelerating potential V



In this experiment, on drawing different I–ϕ curve intensity maxima is obtained at an angle of diffraction of 50° and accelerating potential 54 volt.

Calculations and result:- The appearance of the peak in a particular direction is due to the constructive interference and it is called diffraction maxima. This indicates that moving electrons are behaving like wave. Hence wavelength of these can be calculated by following two methods:

Considering moving electrons as particle

Using de-Broglie formula

$\lambda_e = \frac{12.27}{\sqrt{V}} = \frac{12.27}{\sqrt{54}}$

λ_e = 1.67 Å

Considering moving electrons as wave

Using Bragg's equation

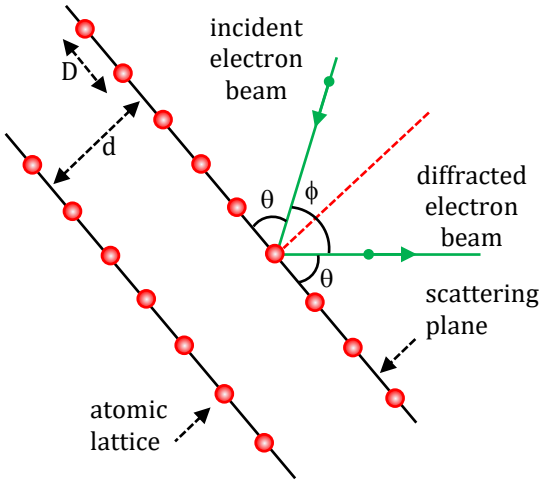
$2d \sin\theta = n\lambda$

Where, $\theta = \frac{180-\phi}{2}$ = 65° & d = 0.91Å for Ni

n = 1 and λ_e = 1.65 Å

Both calculated and experimental wavelengths are equal hence verifies the concept of matter waves (moving particles shows wave character).

Relation between θ and ϕ : $\phi = 180° - 2\theta \Rightarrow \theta = 90° - \frac{\phi}{2}$



Explanation of Bohr Quantisation Condition

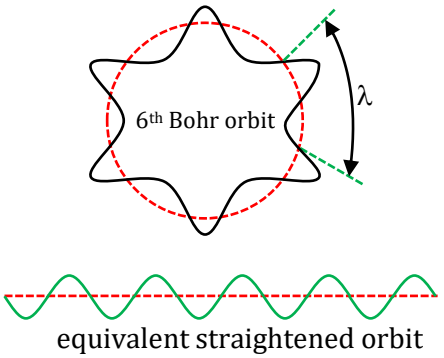
According to De Broglie electron revolves round the nucleus in the form of stationary waves (i. e. wave packet) in the similar fashion as stationary waves in a vibarting string. Electron revolves in those circular orbits whose circumference is an integral multiple of de–Broglie wavelength associated with the electron,

$2\pi r = n\lambda$

$\because \quad \lambda = \frac{h}{mv}$ and $2\pi r = n\lambda$

$\therefore \quad mvr = \frac{nh}{2\pi}$

This is the Bohr quantisation condition.



General form, $2\pi r_n = n\lambda$

Where, n = order of the orbit

r_n = radius of the n^{th} Bohr orbit

$r_n = (0.529\ \text{Å})\ \frac{n^2}{Z}$ [Z = atomic number]

Illustration 18:

Calculate de-Broglie wavelength of an e^- revolving in first excited state in hydrogen atom.

Solution:

Given, n = 2 (first excited state) ; Z = 1 (for hydrogen)

$2\pi r_2 = 2\lambda \ \Rightarrow \ \lambda = \pi r_2 = \pi(0.53\text{Å})\frac{(2)^2}{(1)} = 2.116\ \text{Å}$

Illustration 19:

In third orbit of hydrogen atom wavelength of revolving electron is λ. Calculate the circumference of third Bohr orbit.

Solution:

$2\pi r_3 = 3\lambda$

Circumference of 3^{rd} Bohr orbit = 3λ

Heisenberg Uncertainty Principle

Bohr's theory considers an electron as a material particle. Its position and momentum can be determined with accuracy. But, when an electron is considered in the form of wave as suggested by de-Broglie, it is not possible to ascertain simultaneously the exact position and velocity of the electron more precisely at a given instant since the wave extends throghout a region of space.

Therefore, de-Broglie nicely incorporatated the wave particle duality with Heisenberg uncertainity principle.

In 1927, Werner Heisenberg presented a principle known as Heisenberg uncertainity principle which states that : "It is impossible to measure simultaneously the exact position and exact momentum of a body as small as an electron."

The uncertainity in measurement of position, Δx, and the uncertainity in momentum Δp or mΔv, are related by Heisenberg's relationship as : (p = mv, Δp = mΔv)

$\Delta x.\Delta p \geq \frac{h}{4\pi}$ or $\Delta E.\Delta t \geq \frac{h}{4\pi}$

(i) When $\Delta x = 0$, $\Delta p = \infty$ (ii) When $\Delta p = 0$, $\Delta x = \infty$

So, if the position is known quite accurately, i.e., Δx is very small, Δp becomes large and vice-versa.

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BEGINNER'S BOX-3

1. de-Broglie wavelength of an electron, accelerated by potential V is λ . What will be de-Broglie wavelength of the electron which is accelerated by 4V potential ?

2. Find the ratio of de Broglie wavelength of molecules of hydrogen and helium which are at temperatures 27°C and 127°C respectively.

3. A particle of mass M at rest decays into two particles of masses m_1 and m_2 having non zero velocities. Find out the ratio of the de–Broglie wavelengths of the two particles.

4. Find out velocity of an electron so that its momentum is equal to that of photon with a wavelength of λ = 5200Å.

5. Find out voltage applied to an electron microscope to produce electron of wavelength 0.6Å.

6. What is the effective mass of a photon having wavelength λ ? **[AIPMT 2006]**

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BEGINNER'S BOX — ANSWERS KEY

BEGINNER'S BOX-1

1.	Hertz	**2.**	1.6×10^{-24} kg-m/s	**3.**	(a) 2.5×10^{19} (b) 3.33×10^{-8} N		
4.	Violet	**5.**	1.5×10^{34}	**6.**	2×10^{20}	**7.**	9×10^{24}

BEGINNER'S BOX-2

1.	No	**2.**	64 unit	**3.**	0.34 V	**4.**	2V
5.	4λ	**6.**	0.48 μA, 3×10^{12}	**7.**	1.5 X	**8.**	6.8 eV
9.	$3K + h\nu_0$	**10.**	1.7 eV				

BEGINNER'S BOX-3

1.	$\frac{\lambda}{2}$	**2.**	$\lambda_{H_2} : \lambda_{He} = \sqrt{8} : \sqrt{3}$	**3.**	1 : 1	**4.**	1400 m/s
5.	416.6 volt	**6.**	$m = \frac{h}{\lambda c}$				

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